Performance Analysis and Interleaver Structure Optimization for Short-Frame BICM-OFDM Systems

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Abstract—Bit-interleaved coded modulation (BICM), together with orthogonal frequency-division multiplexing (OFDM) signaling, has found its application in many recent wireless standards. BICM-OFDM systems are able to simplify the design of coding and modulation with various information rates for link adaptation. Furthermore, they provide robustness against frequency-selective fading channels. The combination of BICM-OFDM and convolutional code turns out to be attractive as it allows for low complexity and low latency decoding architecture at the receiver. It has been known that the performance of convolutionally coded BICM-OFDM systems depends on the bit interleaver structure, but little attention has been paid for its theoretical performance analysis. In this paper, we develop a novel theoretical expression for approximate BER of convolutionally coded BICM-OFDM that can be used for evaluating the performance of a given bit interleaver structure, and propose a design guideline for its optimization. Numerical comparisons between the optimized interleaver that satisfies our guideline and other random and structured interleavers demonstrate the validity of our design criterion.

Index Terms—Bit-interleaved coded modulation (BICM), convolutional code, frequency-selective fading channel, interleaver design, orthogonal frequency-division multiplexing (OFDM).

I. INTRODUCTION

In order to achieve high data-rate communication with limited spectral resources over wireless fading channels, the use of channel coding with high-order modulation, together with an orthogonal frequency-division multiplexing (OFDM) signaling has become a de facto standard. By applying a suitable channel coding to OFDM, the frequency diversity effect provided by a frequency-selective nature of wireless channels can be achieved without complicated equalization as well as time-domain interleaving.

Bit-interleaved coded modulation (BICM) was first introduced by Zehavi [1], and later analyzed by Caire et al. [2] from an information theoretic viewpoint. Since then, BICM has gained popularity largely due to its simplicity of coding and modulation design as well as its effectiveness over fading channels. In fact, BICM is known to outperform trellis coded modulation [3] over certain fading channels in terms of cut-off rate and bit error rate (BER) [2]. Therefore, the combination of BICM and OFDM has been considered as a promising approach in the recent standards that target both high bandwidth efficiency and high reliability through the link adaptation over frequency-selective fading channels with limited receiver complexity.

In general, the performance of BICM depends on both channel statistics and constituent binary channel codes. If the channel can be assumed to be ergodic and the size of the channel interleaver is sufficiently large such that each of pulse-amplitude modulation (PAM) or quadrature amplitude modulation (QAM) symbols is subject to statistically independent fading, the so-called capacity approaching channel codes such as low-density parity-check (LDPC) and turbo codes are desirable since they can approach ergodic capacity of fading channels [4], [5]. On the other hand, if the channel interleaver does not provide enough diversity as in the case of a short frame transmission, the resulting channel will be characterized as block fading, and in this case the channel coding within the block does not provide any diversity effect in time domain [6], [7]. In such a channel, the frequency selectivity provided by dispersiveness of a channel impulse response and the minimum Hamming distance of a given channel code determine the achievable diversity order [8].

For real-time streaming applications where the latency requirement is given high priority, transmission of a short frame is commonly employed. In such a case, the channel interleaver cannot be applied to multiple OFDM symbols, or equivalently, all the successive OFDM symbols may experience the same channel state. That is, all the OFDM symbols that form a single codeword may fall within a single block in the block fading paradigm. This paper mainly focuses on such a short-frame transmission scenario where only non-ideal symbol and bit interleaving can be applied. In this case, the capacity approaching channel codes may not be necessarily effective, since the performance is dominated by the diversity order of a channel. Consequently, even classical convolutional codes with much lower decoding complexity may be expected to achieve similar performance as long as the minimum distance of the code is greater than or equal to the achievable diversity order [8]. In fact, simulation results demonstrate that the convolutionally coded BICM-OFDM can achieve similar performance to turbo-coded and LDPC-coded BICM-OFDM systems.
in such a scenario [9]. This observation motivates us to further examine and optimize the performance of convolutionally coded BICM-OFDM, especially considering the fact that it can be encoded and decoded with low latency.

In the BICM literature, the analytical model based on a random interleaver introduced in [2], which is called single-input interleaver or S-interleaver [10, Sec. 8.1], is frequently adopted. On the other hand, the random interleaver introduced in [1] is called multiple-input interleaver, or M-interleaver [10, Sec. 8.1], and its achievable rate and error exponent have been analyzed in [11]. Since the insertion of S-interleaver can average out the unequal error protection (UEP) property introduced by high-order modulations, the model based on S-interleaver significantly simplifies an analysis over AWGN channels without fading [12]–[14] as well as with fading [15], [16]. Nevertheless, whether fading is present or not, the performance of convolutionally coded BICM system considerably depends on the structure of bit interleaver, thus leaving a room for optimization.

An interleaver optimization of BICM over an AWGN channel has been conducted in [17]. Based on the numerical results, it has been shown in [18] that the performance of BICM is significantly improved over an AWGN channel if the interleaver is perfectly removed. This improvement was later analyzed from a theoretical viewpoint in [19], where the distribution of L-values (log-likelihood ratio) introduced in [13] was exploited. Furthermore, it has been proved in [20] that BICM without any interleaver will be optimal for convolutional codes over an AWGN channel in asymptotically high SNR and an insertion of interleaver itself results in suboptimal performance.

Unlike the case of an AWGN channel, interleaving is necessary for frequency-selective fading channels to achieve frequency diversity gain and no interleaving results in significant performance loss. The optimal design of an interleaver for such fading channels has been examined in [21], [22] in conjunction with a single-carrier based BICM system. In most practical wireless systems, however, BICM is concatenated with OFDM signaling. For BICM-OFDM system, the interleaver design that guarantees a full diversity order based on a finite-depth interleaver has been presented in [8], but without exploiting the UEP property. Heuristic approaches that take into account the UEP property of high-order QAM are also proposed in [9], [23]. To the best of the authors’ knowledge, however, an interleaver optimization based on theoretical BER analysis has not been performed for such a system.

In this paper, we derive a novel form of approximate BER for the convolutionally coded BICM-OFDM system. The obtained expression that fully takes into account the UEP property can be used to elucidate the performance difference provided by specific interleaver structures. Based on this result, we also present a design guideline for an interleaver optimization under the practical assumption that an interleaver has a finite depth. Since our primary objective in this paper is to develop a low latency and low complexity BICM-OFDM system, we do not address the issues of BICM with iterative decoding (BICM-ID) or an optimization based on the channel condition with feedback [24], [25].
Let $\mathcal{N}_c = \{1, 2, \ldots, N\}$, $\mathcal{N}_b = \{1, 2, \ldots, m\}$, and $\mathcal{N}_x = \{1, 2, \cdots, K\}$ denote the sets of indices that represent bit positions within each codeword, bit positions within each M-PAM symbol, and subcarrier positions of each OFDM symbol, respectively. The bit interleaver can be considered as a one-to-one reordering mapping:

$$\psi : n \rightarrow (k, i),$$

where $n \in \mathcal{N}_c$ denotes the original ordering of the coded bit $c_n$, which is mapped onto the $i$th bit of the $k$th PAM symbol $x_k$. Here, $k \in \mathcal{N}_x$ is the time ordering of the allocated PAM symbol (i.e., the subcarrier index for our short-frame BICM-OFDM system) and $i \in \mathcal{N}_b$ is its bit position. The specific deterministic and random interleavers used in this work are described in Section IV.

B. Channel Model

We consider a frequency-selective fading channel with $L$ taps represented by $h = [h_1, h_2, \ldots, h_L]^T$ where each element is assumed to be statistically independent and modeled as a zero-mean complex Gaussian random variable with unit variance. The channel impulse response $h$ is assumed to be invariant during the transmission of one OFDM symbol, but it is assumed statistically independent for each OFDM symbol transmission. Therefore, the channel model can be considered as a frequency-selective and block fading channel. A cyclic prefix (CP) is added to each OFDM symbol before transmission. We assume that the length of CP is long enough such that the effect of the inter-symbol interference (ISI) associated with a channel delay spread is negligible.

C. Receiver

After the CP is removed and FFT operation is performed at the receiver, the $k$th subcarrier of the received OFDM symbol $y$ is expressed as

$$y_k = H_k x_k + n_k$$

where $x_k$ is the transmitted symbol on the $k$th subcarrier and $n_k$ is a zero-mean complex AWGN term with variance $N_0/2$ per dimension. The channel coefficient of the $k$th subcarrier $H_k$ is given by

$$H_k = W_K H(k) P h$$

where $W_K(k) = [1, W_k^1, W_k^2, \ldots, W_k^{(L-1)K}]$ is an $L \times 1$ column vector consisting of $W_K \triangleq e^{-j2\pi/K}$, with the superscript $(\cdot)^H$ indicating a conjugate transpose of given matrices or vectors, and $P$ is an $L \times L$ diagonal matrix with $p_l$, for $l = 1, 2, \ldots, L$, on its main diagonal. Each of the $p_l$ takes a non-negative value and they represent the power delay profile (PDP) of the $L$-tap frequency-selective channel. Note that without loss of generality both the transmitted and received symbols are assumed to have an average energy of unity and thus $\sum_{l=1}^{L} p_l^2 = 1$.

At the receiver side, we calculate the bit metric vector $\lambda'$ of the entire coded and interleaved bit sequence. A set of bit metrics is then permuted by a bit deinterleaver $\Pi^{-1}$ to retrieve that in the original order, denoted by $\lambda$. Then the bit metric of the $n$th coded bit $c_n$ with the perfect channel state information (CSI) can be given by

$$\lambda_n = \max_{x \in \mathcal{X}_k} \log f(y_k|x, H_k), \quad \text{for} \quad n = \psi^{-1}(k, i),$$

where $\psi^{-1}$ is an inverse mapping of $\psi$ in (1), $x$ is a candidate of transmitted symbol, $\mathcal{X}_k$ is the subset of all the PAM constellation points with its $i$th bit position specified by the value $b \in \{0, 1\}$, and $f(y_k|x, H_k)$ is the conditional probability density function (PDF) that can be expressed as

$$f(y_k|x, H_k) = \frac{1}{\pi N_0} \exp \left(-\frac{|y_k - H_k x_k|^2}{N_0}\right).$$

The Viterbi decoder determines the maximum likelihood (ML) path (i.e., the most likely coded sequence) $\hat{c}$ according to the following rule:

$$\hat{c} = \arg \max_{\mathcal{C}} \left\{ \sum_{n=1}^{N} \lambda_n \right\}$$

where $\mathcal{C}$ is a candidate codeword and $C$ is the code.

III. BER PERFORMANCE ANALYSIS

A. Union Bound Analysis

We start with a brief review of the standard union bound analysis for binary convolutional codes. The union bound serves as a general upper bound for evaluating BER performance, which for convolutional codes is given by [27, Sec. 4.4]

$$P_b \leq \frac{1}{k_c} \sum_{d_l = d_l}^{\infty} \beta(d_l) P(d_l)$$

where $d_l$ is the minimum free distance of the convolutional codes, $\beta(d_l)$ denotes the total input weight of the error events with the Hamming distance $d_l$, and $P(d_l)$ denotes the codeword pairwise error probability (PEP) with the Hamming distance $d_l$. Moreover, $k_c$ is the number of the information bits per trellis segment, and it is equal to one throughout this work since we assume the use of the rate-1/n_c convolutional codes for simplicity. Extension of our analysis to other code rate cases is straightforward.

The union bound generally provides a tight upper bound in high SNR region, but its exact calculation involves prohibitively high computational complexity. Since the contribution of error events decays exponentially with their Hamming distances, one approximate approach is to evaluate only the error event with the minimum free distance $d_l$ (i.e., the shortest error event). This will then serve as a lower bound, given by

$$P_b \geq \frac{1}{k_c} \beta(d_l) P(d_l).$$

As SNR increases, the bound (8) becomes tight since the error event with $d_l$ is dominant in all the error events. For conventional binary convolutional codes, $\beta(d_l)$ can be readily obtained from the weight enumerator of codes and those of the optimal codes recently found are tabulated in [28]. Thus, we focus on calculating the codeword PEP $P(d_l)$ in the framework of BICM-OFDM throughout this paper.
Due to the linearity of convolutional codes, assumption of the all-zero codeword transmission is sufficient for calculating the codeword PEP $P(d_0)$ if a channel is symmetric and memoryless. However, a UEP property of BICM does not preserve the symmetry property in general. An exception is the case of S-interleaver [10, Sec. 8.1], where the UEP property is averaged out. In such a case, the insertion of S-interleaver is equivalent to a symmetrization of channels based on random labeling introduced in [2], where the label $\mu$ and its complementary label $\bar{\mu}$ are chosen with equal probability for each coded bit.

Since our objective in this work is to analyze the BICM performance for a given specific interleaver structure, it is necessary to deal with the UEP property by computing the distribution of the squared Euclidean distance for all possible symbol pairs [16]. Specifically, we assume that the all-zero codeword is transmitted, but we compute the distribution of the squared Euclidean distance between the reference symbols (that represent the all-zero codeword) and other possible symbols.

### B. Expression for PEP

Let us consider the specific codeword PEP $P(\mathbf{c} \rightarrow \hat{\mathbf{c}})$ that the transmitted codeword $\mathbf{c} = [c_1, \ldots, c_N]$ is decoded at the receiver in favor of another codeword $\hat{\mathbf{c}} = [\hat{c}_1, \ldots, \hat{c}_N]$, where $\mathbf{c}$ and $\hat{\mathbf{c}}$ differ by the minimum free distance $d_0$.

Let $N(\mathbf{c}, \hat{\mathbf{c}})$ denote the set of the indices $n$ of the coded bits where $c_n \neq \hat{c}_n$. By assumption it follows that $|N(\mathbf{c}, \hat{\mathbf{c}})| = d_0$, where $|S|$ denotes the cardinality of an arbitrary set $S$. Furthermore, let $n = [n_1, n_2, \ldots, n_{d_0}] \in N(\mathbf{c}, \hat{\mathbf{c}})$ denote the vector of the elements $n \in N(\mathbf{c}, \hat{\mathbf{c}})$ that are sorted in an increasing order, i.e., $n_1 < n_2 < \cdots < n_{d_0}$. Let $i_w$ and $k_w$ denote the corresponding outputs of the mapping function (1) with its input given by $n_w$, where $w \in \{1, 2, \ldots, d_0\}$ represents the error bit index in this specific error event. We assume that the $d_0$ error bits are mapped onto different PAM symbols, i.e., the $k_w$ for $w \in \{1, \ldots, d_0\}$ are distinct. We further define $X = [x_{k_1}, x_{k_2}, \ldots, x_{k_{d_0}}] \in X^{d_0}$ and $S = [i_{w1}, i_{w2}, \ldots, i_{wd_0}] \in X^{d_0}$, where the former represents the PAM symbol vector and the latter the corresponding bit position vector, both associated with the bit error position vector of the codeword $n$. Note that the vector $S$ depends on the particular code structure of the convolutional code and the interleaver structure. The purpose of the interleaver optimization is thus to control the distribution of $S$ such that the resulting bit error rate is minimized. The vector $X$ can be characterized as a deterministic variable once the complete interleaving pattern is specified. Nevertheless, since each element $x_{k_w}$ of the PAM symbol vector $X$ depends not only on the corresponding coded bit $c_{n_w}$ but also on the other coded bits that constitute the symbol, the vector $X$ will be characterized as a random vector that depends on the specific pattern of $S$.

Example 1: Consider the 4-PAM constellation (i.e., $m = 2$) with its labeling shown in Fig. 2. Suppose that we employ the convolutional code with the constraint length $K_c = 3$ and the code rate $r_c = 1/2$ whose generator polynomial is given by (5,7) in octal, where its trellis diagram is sketched in Fig. 3. In this case, we have the minimum free distance $d_0 = 5$ [28], and an element of $S$ is $i_w \in \{1, 2\}$. Let us now assume that the interleaver is stochastic and thus determined in a probabilistic manner. Then it is reasonable to assume that each $i_w$ in $S$ takes the value from $\{1, 2\}$ with equal probability. Therefore, $S$ is a random vector that has the uniform distribution where its realization is chosen from the set $\{[1, 1, 1, 1, 1], [1, 1, 1, 1, 2], \ldots, [2, 2, 2, 2, 2]\}$ with equal probability of $1/2^5$. On the other hand, in this particular scenario, one can design the deterministic interleaver where $S$ is always chosen from the set where each element contains three 1’s and two 2’s, e.g., $\{[2, 2, 1, 1, 1], [2, 1, 2, 1, 1], \cdots, [1, 1, 1, 2, 2]\}$, as will be described later in Example 2.

Given the above expressions, the metric difference between $\mathbf{c}$ and $\hat{\mathbf{c}}$ can be expressed as

$$\delta = \sum_{n \in N(\mathbf{c}, \hat{\mathbf{c}})} \{\chi_n^a - \chi_n^{\bar{a}}\} = \sum_{w=1}^{d_0} \left\{\max_{x \in X^{d_0}} \log f(y_{k_w}, x, H_{k_w}) - \max_{x \in X^{d_0}} \log f(y_{k_w}, x, H_{k_w})\right\},$$

where $\bar{a}$ represents the binary complement of a given bit $a$. Since we assume that the binary convolutional codes are employed, $\hat{c}_n$ equals $\bar{c}_n$ and thus $\chi_n^a$ equals $\chi_n^{\bar{a}}$.

Given $X$, $S$, and the channel coefficient vector $H = \begin{bmatrix} 2d_{\min} \end{bmatrix}$ with 4-PAM constellation with BRGC labeling. The underlined bits represent the first position in a symbol (i.e., $i = 1$).
where the expectation is over $\mathbf{H}$ and the random variables
\[ \Delta_w (i) = |x_{k_w} - x'_{k_w} (i)|^2, \quad (16) \]

the distribution of which depends on the vector $\mathbf{s}$.

Note that $\Delta_w (i)$ in (16) represents the squared Euclidean distance between the transmitted symbol $x_{k_w}$ and its nearest neighbor symbol with the flipped bit from $x_{k_w}$ at the bit position $i$. Once $x_{k_w}$ is determined from $c_{n_w}$ and $i$, the corresponding complement $x'_{k_w} (i)$ associated with $c_{n_w}$ and $i$ is uniquely determined. Consequently, the distribution of $\Delta_w (i)$ becomes independent of the coded bit value $c_{n_w}$ and thus depends only on the bit position $i$.

For example, in the case of the 4-PAM constellation illustrated in Fig. 2, the probability distribution of $\Delta_w (i)$ conditioned on the bit position $i$ is expressed by enumerating the squared Euclidean distance for all pairs of $x_{k_w}$ and $x'_{k_w} (i)$ as
\[ P_{\Delta} (l | i = 1) = \Pr (\Delta = l d_{\text{min}}^2 | i = 1) = \frac{1}{2} \delta_{l,1} + \frac{1}{2} \delta_{l,4} \quad (17) \]
\[ P_{\Delta} (l | i = 2) = \Pr (\Delta = l d_{\text{min}}^2 | i = 2) = \delta_{l,1}, \quad (18) \]

where $\delta_{a,b}$ is the Kronecker delta function and $d_{\text{min}}$ represents the minimum Euclidean distance of the constellation. In the above expression, we implicitly assume that once the bit position $i$ is specified, the distribution of $\Delta_w (i)$ does not depend on the particular value of the error bit index $w$.

Upon computation of the probability distribution, it turns out to be convenient to work in its $z$-transform domain, i.e., the probability generating function defined as
\[ F_{\Delta} (Z) = \sum_{l=0}^{\infty} P_{\Delta} (l) Z^l \quad (19) \]

where $Z$ is an auxiliary variable. With this expression, (17) and (18) can be re-expressed as
\[ F_{\Delta} (Z) | i = 1) = \frac{1}{2} Z + \frac{1}{2} Z^4 \quad (20) \]
\[ F_{\Delta} (Z) | i = 2) = Z \quad (21) \]

D. BER Expression for an AWGN Channel

In the case of an AWGN channel, $H_{k_w}$ can be replaced by unity and thus the conditional PEP $P(d_l | S)$ is expressed as
\[ P(d_l | S) \approx E_{\Delta_{\text{sum}}} \left\{ Q \left( \sqrt{\frac{\Delta_{\text{sum}}}{2 N_0}} \right) \right\}, \quad (22) \]

where $\Delta_{\text{sum}} = \sum_{w=1}^{d_i} \Delta_w (i_w)$ for a given $S$. Under the assumption that the $\Delta_w (i)$ are statistically independent, the probability distribution of $\Delta_{\text{sum}}$ conditioned on $S$ is expressed as
\[ P_{\Delta_{\text{sum}}} (l | S) = \Pr (\Delta_{\text{sum}} = l d_{\text{min}}^2 | S) \quad (23) \]

is given by
\[ P_{\Delta_{\text{sum}}} (l | S) = P_{\Delta} (l | i_1) \ast \cdots \ast P_{\Delta} (l | i_{d_i}) \quad (24) \]
where \( * \) denotes a convolution operation. The above probability distribution should be expressed in the form as

\[
P_{\Delta, \text{sum}}(\| \mathcal{S} \|) = \sum_{j=1}^{\Gamma} P_j(\mathcal{S}) d_l A_j
\]

where \( \Gamma \) denotes the number of possible distinct values of \( \Delta, \text{sum} \), \( A_j \) denotes its \( j \)th value, and \( P_j(\mathcal{S}) \) is the corresponding conditional probability, i.e.,

\[
P_j(\mathcal{S}) = \Pr(\Delta, \text{sum} = A_j d^2_{\min} | \mathcal{S} ).
\]

Alternatively, using \( z \)-transform expression, one may replace the convolution of (24) by the product and thus it can be simplified as

\[
F_{\Delta, \text{sum}}(Z|\mathcal{S}) = \prod_{w=1}^{d_l} F_{\Delta}(Z|w) = \sum_{j=1}^{\Gamma} P_j(\mathcal{S}) Z^A_j.
\]

Once the set of \( P_j(\mathcal{S}) \) can be found through the above process, the conditional PEP of (22) can be rewritten as

\[
P(d_l|\mathcal{S}) \approx \sum_{j=1}^{\Gamma} P_j(\mathcal{S}) Q\left( \sqrt{\frac{A_j d^2_{\min}}{2N_0}} \right) \geq P_{\min}(\mathcal{S}) Q\left( \sqrt{\frac{A_{\min} d^2_{\min}}{2N_0}} \right)
\]

where

\[
A_{\min} = \min_{1 \leq j \leq \Gamma} A_j
\]

and \( P_{\min}(\mathcal{S}) \) is the probability of (26) associated with \( A_{\min} \). When SNR is high, since the \( Q \) values associated with the terms other than the minimum value \( A_{\min} d^2_{\min} \) is negligibly small compared to that with \( A_{\min} d^2_{\min} \), the lower bound of (28) becomes tight.

We further note that in the case of BRGC labeling, the value of \( A_{\min} \) cannot be made smaller than \( d_l \) regardless of the interleaver structures since the conditional distribution \( F_{\Delta}(Z|i) \) for any bit position \( i \) with BRGC labeled constellation has the term \( Z \) as its minimum degree and thus \( F_{\Delta, \text{sum}}(Z|\mathcal{S}) \) for any realization of \( \mathcal{S} \) must have \( Z^{d_l} \) as its smallest degree term. More specifically, we have

\[
F_{\Delta}(Z|i) = p_{\min}(i) Z + \text{higher degree terms}
\]

where we define

\[
p_{\min}(i) = P_{\Delta}(l = 1|i),
\]

and thus

\[
F_{\Delta, \text{sum}}(Z|\mathcal{S}) = \prod_{w=1}^{d_l} p_{\min}(i_w) Z^{d_l} + \text{higher degree terms}.
\]

Therefore, we always have \( A_{\min} = d_l \) at the best and the lower bound of (28) is given by

\[
P(d_l|\mathcal{S}) \geq P_{\min}(\mathcal{S}) Q\left( \sqrt{\frac{d_l d^2_{\min}}{2N_0}} \right)
\]

where

\[
P_{\min}(\mathcal{S}) = \prod_{w=1}^{d_l} p_{\min}(i_w).
\]

By taking an average over \( \mathcal{S} \), we may express the approximate lower bound for the unconditional PEP as

\[
P(d_l) \geq P_{\min} Q\left( \sqrt{\frac{d_l d^2_{\min}}{2N_0}} \right)
\]

where

\[
P_{\min} = E_{\mathcal{S}} \{P_{\min}(\mathcal{S})\}
\]

when \( \mathcal{S} \) is stochastic. On the other hand, if \( \mathcal{S} \) is deterministic, we have

\[
P_{\min} = \sum_{\mathcal{S}} \frac{A(\mathcal{S})}{A_{\text{total}}} P_{\min}(\mathcal{S}),
\]

where \( A(\mathcal{S}) \) is the number of the error events that correspond to \( \mathcal{S} \), and

\[
A_{\text{total}} = \sum_{\mathcal{S}} A(\mathcal{S}).
\]

Finally, by substituting (35) into (8) we can obtain the following approximate BER lower bound:

\[
P_b \geq \frac{1}{K_c} \beta(d_l) P_{\min} Q\left( \sqrt{\frac{d_l d^2_{\min}}{2N_0}} \right).
\]

Note again that the distribution of \( \mathcal{S} \) depends on the interleaver structures, and our main objective in this paper is to examine the performance difference provided by specific interleaver structures. Therefore, it is important to derive the distribution of \( \mathcal{S} \) for a given interleaver.

E. Proposed Interleaver Design Guideline

From the unconditional PEP lower bound expression of (35), together with (37), our optimization criterion is to minimize

\[
\sum_{\mathcal{S}} A(\mathcal{S}) P_{\min}(\mathcal{S}).
\]

Now, the following observation can be made; in the case of the PAM constellation with BRGC labeling, we may have

\[
p_{\min}(i) = 2^{-(m-i)}, \quad i = 1, 2, \cdots, m,
\]

where \( m \) represents the number of bits that form one PAM symbol, and as \( i \) increases toward \( m \), \( p_{\min}(i) \) also increases. From this observation and (34), it follows that the vector \( \mathcal{S} \) that contains smaller values of \( i \) as majority of its elements may achieve a lower value of \( P_{\min}(\mathcal{S}) \), thus contributing to lower error rate. We refer to a bit position with smaller \( i \) as a stronger bit position in what follows. The bit position strength is explicitly related to uncoded error rate and mutual information [10, Sec. 8.1]. The expression (34) can be alternatively expressed as

\[
P_{\min}(\mathcal{S}) = \prod_{i=1}^{m} p_{\min}^{B_i}(\mathcal{S}) = \prod_{i=1}^{m} 2^{-B_i(\mathcal{S})(m-i)},
\]
where $B_i(S)$ is the number of elements $i$ in the vector $S$. Therefore, if $B_i(S)$ is large for small $i$, the resulting error rate can be made small.

We describe the interleaver design process as follows. Let $s = [i_1, i_2, \ldots, i_N]$ denote the bit position vector corresponding to the *entire* codeword $c$ where $i_n \in N_b$ represents the bit position onto which $c_n$ is mapped. There is the obvious constraint on $s$ that the number of the bits mapped onto each bit position is equal. Thus, by $T_i$, representing the number of bits mapped onto the $i$th bit position in the entire codeword, we must have $T_1 = T_2 = \cdots = T_m = K$. Furthermore, let $\varepsilon_k(n) = [\varepsilon_{n+1}, \varepsilon_{n+2}, \ldots, \varepsilon_{n+\tau}] \in \{0, 1\}^\tau$ denote the $k$th error event vector that represents the divergent path from the correct path at the $(n+1)$th bit with the Hamming weight of $d_i$, where $\tau$ represents the bit length of the divergent path. (Note that $k$ is the index of the error event and if only the single error event with the minimum Hamming weight of $d_i$ exists at each trellis section for a given convolutional code, then we only have $k = 1$.) For a given pair of $s$ and $\varepsilon_k(n)$, the vector $\hat{s}$ is uniquely determined. Therefore, in order to make this dependence explicit, we rewrite $s$ as $\hat{s}[\varepsilon_k(n)]$ wherever it is appropriate.

Noticing the fact that for the rate-$1/n_c$ convolutional codes, the divergent path should occur at the positions $n = 1, 1 + n_c, \ldots$, we have the the following optimization problem:

$$
\min_{\hat{s}} \sum_k \sum_{j=0}^{(N-\tau)/n_c} P_{\min}[\hat{s}[\varepsilon_k(jn_c)]] \quad (43)
$$

subject to $T_i = K$ for $i = 1, 2, \ldots, m$. \hspace{1em} (44)

In this work, we will perform the brute-force search to solve the above problem for small $N$, and denote the resulting optimal bit position order by $s_{\text{opt}}$. Apparently, the brute-force search for large $N$ requires significantly high computational complexity and soon becomes unrealistic. Therefore, we will initially resort to the brute-force search in the following example for small $N$ in order to examine the general properties of $s_{\text{opt}}$, which will be then exploited for the case of large $N$.

**Example 2:** We continue the scenario of Example 1; we consider the 4-PAM constellation shown in Fig. 2 and the rate-$1/2$ convolutional code with its trellis diagram depicted in Fig. 3. In this case, we have $i \in \{1, 2\}$ where the value $i = 1$ corresponds to the stronger position than $i = 2$. As shown in Fig. 3, there exists only one error pattern associated with the minimum free distance $d_t = 5$, which is represented as $\varepsilon_{01}(0) = [1, 1, 0, 1, 1, 1]$ with $\tau = 6$. If the codeword length is 20, we obtain $s_{\text{opt}} = [2, 2, 2, 1, 1, 1, 2, 1, 2, 1, 2, 1, 1, 2, 2, 1, 2, 2]$ by solving (43) with the constraint (44) through the brute-force search. In fact, by inspecting this optimal sequence, we observe that all the eight possible error vectors $S$ are of the form $[2, 2, 1, 1, 1], [2, 1, 1, 2, 1, \ldots]$, i.e., they all satisfy $B_1(S) = 3$ and $B_2(S) = 2$, which is the most desirable property in view of (42).

The above example illustrates that the best interleaver should depend on the patterns of the error events with the minimum free distance. We also observe that except for the head and tail of the trellis (i.e., the first and the last six bits), the bits of the optimal vector are arranged such that 2 (odd positions) and 1 (even positions) alternately appear, and this agrees with the fact that if we compare the number of 1’s in the odd and even positions of the error event vector $e_k(n)$, that in the even positions is larger than that in the odd positions. In other words, considering the constraint (44), the number of bit positions should be well balanced but we also observe that the positions with a larger number of 1’s in the error event vector should be matched with the stronger bit positions in a PAM symbol.

In summary, as the design guideline for optimal interleavers, we impose the following two properties:

**P1** The bit position $i$ should be arranged in $s$ such that the stronger and weaker bit indices are well balanced. (That is, except for the edges, the trains of 1’s (or 2’s) should be avoided as observed in Example 2.)

**P2** The places of the strong and weak bit positions should be ordered according to the error event with the minimum free distance. (That is, the bit position order of [1, 2] and [2, 1] makes difference in Example 2.)

We note that the properties P1 and P2 are also reported in [23] and [17], respectively. In this work, we will focus on the interleaver optimization under the finite-depth interleaving based on the above design guideline in combination with an additional guideline associated with frequency diversity gain described in the following subsection. Furthermore, we will investigate the contribution of each property to the performance improvement in the subsequent sections.

**F. Fading Channel**

The most important role of the interleaver in BICM-OFDM systems over frequency-selective fading channels is to achieve the frequency diversity in addition to the increase of the Euclidean distance $\Delta_{\text{num}}$ discussed in the previous subsection. The amount of achievable diversity effect is associated with the statistical independence among the $d_t$ values of $H_{k\nu}$ in (15). In other words, full diversity can be achieved when the $H_{k\nu}$ are uncorrelated and, conversely, the diversity effect may not be exploited if they have strong dependence.

In this subsection, we will derive an approximate BER of BICM-OFDM system with a specific interleaver over a frequency-selective fading channel under the assumption that the full diversity gain is achieved. We note that our subsequent derivation inherits the analytical approach and notations developed in [8]. Nevertheless, since the main focus of [8] is on the achievable diversity order, it does not address the UEP property. On the other hand, our analytical modeling takes into account the UEP property and thus is able to encompass more general deterministic interleavers.
Based on the expression (15) and using (3), we may write
\[
\frac{d}{w=1} |H_{kw}|^2 \Delta_w (i_w) \\
= \sum_{w=1}^{d} h^H P W_K (k_w) W_K^H (k_w) P h \Delta_w (i_w) \\
= \sum_{w=1}^{d} h^H P \sum_{k=1}^{d} W_K (k_w) W_K^H (k_w) \Delta_w (i_w) \\
= \frac{h^H P}{h} \sum_{w=1}^{d} A (k_w) \Delta_w (i_w) \\
= \frac{h^H P}{A'} \sum_{w=1}^{d} A (k_w) \Delta_w (i_w) \\
\]
where $A'$ and $A (k_w)$ are $L \times L$ matrices, $A' = \sum_{w=1}^{d} A (k_w) \Delta_w (i_w)$, and $A (k_w) = W_K (k_w) W_K^H (k_w)$. We define an $L \times L$ matrix $A$ as $A = \sum_{w=1}^{d} A (k_w)$. Since the rank of the matrix $A$ can be given by $\text{rank}(A) = \text{min}(d, L)$ [8], we also have $r$, the rank of the matrix $A'$, given by $r = \text{rank}(A') = \text{min}(d, L)$. Since $P$ is a non-singular diagonal matrix, the rank of $PA'P$ is also equal to $r = \text{min}(d, L)$. Moreover, since $A (k_w)$, $A'$, and $PA'P$ are positive semi-definite Hermitian, the singular value decomposition (SVD) of $PA'P$ can be expressed as
\[
PA'P = V \Lambda V^H 
\]
where $V$ is an $L \times L$ unitary matrix and $\Lambda$ is an $L \times L$ diagonal matrix with its eigenvalue, $\{\lambda_{\ell} (PA'P)\}_{\ell=1}^{r}$ in decreasing order, on the main diagonal. Since the rank of $PA'P$ is $r = \text{min}(d, L)$, there exist only $r$ nonzero eigenvalues in $\{\lambda_{\ell} (PA'P)\}_{\ell=1}^{r}$. Thus, the conditional PEP can be approximated by
\[
P(\xi \to \xi | X, S, H) \approx Q \left( \frac{\sum_{\ell=1}^{r} \lambda_{\ell} (PA'P) |v_{\ell}|^2}{2N_0} \right) 
\]
where $v_{\ell}$ is the $\ell$th element of the vector $V^H \lambda$ for $\ell = 1, 2, \ldots, L$, and its amplitude follows Rayleigh distribution with its PDF given by $2|v_{\ell}|^2 e^{-|v_{\ell}|^2}$.

Assuming that the full diversity condition is achieved by interleaving such that each subcarrier index $k$ is separated by sufficient distance, the sum of all the nonzero eigenvalues $\{\lambda_{\ell} (PA'P)\}_{\ell=1}^{r}$ can be replaced by the sum of $\Delta_w (i_w)$. In the case that $d_1$ is smaller than $L$ (i.e., $r = d_1$), this replacement can be represented by
\[
\sum_{\ell=1}^{r} \lambda_{\ell} (PA'P) = \sum_{w=1}^{d_1} \Delta_w (i_w). 
\]
On the other hand, in the case that $L$ is smaller than $d_1$, we should consider $\binom{d_1}{L}$ combinations in $\{\Delta_w (i_w)\}_{w=1}^{d_1}$. In this paper, we assume the former case to make our analysis tractable. In this case, the conditional PEP can be written as
\[
P(\xi \to \xi | X, S, H) \approx Q \left( \frac{\sum_{w=1}^{d_1} |v_w|^2 \Delta_w (i_w)}{2N_0} \right) \\
= \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( -\frac{\sum_{w=1}^{d_1} |v_w|^2 \Delta_w (i_w)}{4N_0 \sin^2 \theta} \right) d\theta. 
\]
By taking expectation with respect to $H$ (i.e., $v_w$), (49) can be rewritten as
\[
P(\xi \to \xi | X, S) = E_{\|H\|} \left[ P(\xi \to \xi | X, S, H) \right] \\
\approx \frac{1}{\pi} \int_{0}^{\pi/2} \prod_{w=1}^{d_1} \left( 1 + \frac{1}{4N_0 \sin^2 \theta} \right) d\theta. 
\]

Finally, by following the similar process to the AWGN case and taking an average with respect to $S$, the approximate lower bound of the BER $P_{\text{th}}$ can be obtained. This lower bound can be used to elucidate the performance difference provided by specific interleaver structures as demonstrated in Section V.

In general, the fading coefficient $H_{kw}$ for $w \in \{1, \ldots, d_1\}$ should not be strongly correlated in order to exploit the full diversity effect offered by a channel. Therefore, the interleaver should satisfy the condition that the $d_1$ channel coefficients corresponding to the erroneous subcarriers are separated with the maximum spacing such that the correlation among them becomes negligible.

IV. SPECIFIC INTERLEAVERS AND THEIR PROPERTIES

In practice, the bit interleaver should have a simple deterministic block-wise structure that can be easily implemented. Nevertheless, the interleavers with statistical randomness are also considered in the literature, mainly for its analytical tractability. In this work, we take both the cases into account for our analysis. Specifically, in this section, we will describe the three classes of bit interleavers referred to as random, regular block, and optimized block, in the framework of BICM-OFDM systems and discuss their properties in contrast to our design guideline developed in Section III-E. Moreover, we will formulate the probability $P_{\text{min}}$ that determines the performance of the interleavers and also required to calculate the approximate BER lower bound developed in the previous section.
A. Random Interleaver

As a random interleaver, we consider S-interleaver [10, Sec. 8.1] in what follows, where the bit position $i_w$ is assumed as a random variable with its value chosen from all bit position indices with equal probability. Thus, $S$ has the distribution such that its realization is chosen from the set \{1, \ldots, 1, 1, \ldots, 2, \ldots, [m, \ldots, m, m]\} with equal probability, where each vector has $d_i$ elements. A simple example in the case of 4-PAM and convolutional codes with $d_i = 5$ is discussed in Example 1 in the previous section.

The distribution of $\Delta_{\text{sum}}$ in the $z$-transform domain can be expressed as

$$F_{\Delta_{\text{sum}}}(Z) = \left\{\frac{1}{m} \sum_{i=1}^{m} F_{\Delta}(Z|i)\right\}^{d_i}. \quad (52)$$

It follows that, with $p_{\text{min}}(i)$ denoting the coefficient of the term $Z$ in the polynomial $F_{\Delta}(Z|i)$, $P_{\text{min}}$ can be calculated by

$$P_{\text{min}} = \left\{\frac{1}{m} \sum_{i=1}^{m} p_{\text{min}}(i)\right\}^{d_i}. \quad (53)$$

B. Regular Block Interleaver

Let $(N_r, N_c)$ denote the block structure for the regular and optimized block interleavers described hereafter, where $N_r$ and $N_c$ represent the numbers of rows and columns, respectively. We refer to the interleavers with this block structure as $N_r \times N_c$ interleavers.

The regular block interleaver stores coded bits in the row direction and reads in the column direction to permute bits at the transmitter side. Due to its block structure, it can provide a certain diversity effect since consecutive bits in the original order are separated exactly by $N_r$ bits and mapped onto the distinct and well separated symbols. However, this interleaver may include the cases where the weaker bits are arranged as consecutive outputs. For example, this is caused when the number of rows $N_c$ equals a multiple of $m$ [9], with $m$ representing the number of bits constituting one PAM symbol. In this case, the bit position order of the deinterleaved coded bits is represented as

$$s = \left[\begin{array}{c}
\frac{1}{N_r} \cdot n_i \cdot \text{bits} \\
\frac{1}{N_c} \cdot n_i \cdot \text{bits} \\
\frac{1}{N_r} \cdot n_i \cdot \text{bits} \\
\frac{1}{N_c} \cdot n_i \cdot \text{bits}
\end{array}\right]\quad (54)$$

where each of $mN_c$ bit position indices is formed by $N_c$ consecutive bits of the same bit position index, and they are periodically repeated by $N_r/m$ times. (Note that the total number of bits in $s$ is $mN_c \times N_r/m = N_c N_r = N_r$.) An example of the bit position order in the case of 4-PAM and $72 \times 43$ regular block interleaver is shown in Fig. 4. This bit position order violates our design guideline $\text{P1}$ and in fact it may achieve the maximum value of $P_{\text{min}}(S)$. Therefore, it can be concluded that the regular block interleaver can provide a certain diversity effect but there may exist some cases where the resulting average probability $P_{\text{min}}$ has a relatively large value.

C. Optimized Block Interleaver

An optimal interleaver should follow the design guideline described in the previous section. The optimized block interleaver outlined in [9] is designed such that the variance of the branch metrics in the trellis of the entire codeword is minimized by applying simple row and column vector operations to the regular block interleaver.

We describe the construction of the optimized block interleaver by a specific example, assuming the rate-1/2 convolutional codes in what follows. Recall that for the BRGC labeled $M$-PAM constellation with bit position $i \in \{1, \ldots, m\}$, the bit strength decreases as $i$ increases. In this case, $g$ of the optimized block interleaver has the property that the two bits with $i = 1$ and $i = m$ are paired within one trellis segment and so on such that the variance of the branch metrics is minimized. This satisfies $\text{P1}$ of our design guideline.

With respect to $\text{P2}$, we categorize the optimized block interleaver into the following two types based on the allocation of the weaker bits in $s$: the type-1 optimized block interleavers have the form $s = [m, 1, m - 1, 2, \ldots, \lceil \frac{m}{2} \rceil, m, 1, \ldots]$ where the weaker bits are allocated at the odd positions, and the type-2 optimized block interleavers have the form $s = [1, m, 2, m - 1, \ldots, \lceil \frac{m}{2} \rceil, \lceil \frac{m}{2} \rceil - 1, 1, m, \ldots]$ where the weaker bits are allocated at the even positions. Obviously, it is expected that the type-1 optimized block interleaver works well when the number of zeros that appear in the odd positions of the trellis segments corresponding to the error events with the minimum free distance is greater than that in the even position. Likewise, it is expected that the type-2 optimized block interleaver works well for the opposite case.

We note that in the case of general rate-1/$n_c$ convolutional codes, we can define $n_c$ distinct types of optimized block interleavers based on the allocation of the weak bits.

The optimization process for satisfying our design guideline based on the regular block interleaver is described by the following two steps. In the first step, cyclic row and column shifting is applied to the regular block interleaver in order to obtain the original bit position order (i.e., $s = [1, 2, \ldots, m, \ldots]$). In the second step, by changing the order of mapping to PAM symbols such that the bits of $i = 1$ and $i = m$ are paired within one trellis segment and so on, we can transform the original order into the optimal order. For example, in the case of 4-PAM with BRGC labeling shown
in Fig. 4, we obtain the original order $s = [1, 2, \cdots]$ by cyclically shifting all the elements in the even columns by one bit. Note that the cyclic row shifting is not necessary in this example [9]. Next, by changing the order of mapping, $s = [2, 1, \cdots]$ for the type-1 optimized block interleavers are obtained. (In this 4-PAM example, since the original order is equal to the order of the type-2 optimized block interleaver, the second step is not performed.) The generalized optimization algorithm for $(N_r, N_c)$ block structure and M-PAM constellation is presented in [9].

Note that in the cases of the regular and optimized block interleavers, the distribution of $S$ can be obtained by enumerating them for all possible starting points of the error event, through which the distribution of $\Delta_{\text{sum}}$ can be calculated. The resulting $P_{\text{min}}$ should be directly calculated with reference to (43) as

$$P_{\text{min}} = \sum_k \frac{1}{(N+\tau)/n_c + 1} \sum_{j=0}^{(N-\tau)/n_c} P_{\text{min}}(S[s, e_k (jn_c)]).$$

(55)

V. NUMERICAL RESULTS

In this section, we will numerically evaluate the validity of our BER analysis for both AWGN and frequency-selective fading channels through the comparison of the BER performance. We also examine the effectiveness of our design guideline for interleaver optimization.

In our simulation, we consider the BRGC labeled 4-PAM ($m = 2$) and 16-PAM ($m = 4$) for modulation. For coding, the rate-1/2 convolutional codes with constraint length $K_c = 3$ and 7 are employed, with their generator polynomials given by (5,7) and (133,171) in octal, respectively. The codeword length is chosen as 3096, and $72 \times 43$ block structure for block interleavers is used. Since the number of rows $N_r$ is a multiple of $m$, long runs of weak bits will occur in the regular block interleaver for both the modulation types (cf. Fig. 4). The number of OFDM subcarriers is chosen as 1548 for 4-PAM and 774 for 16-PAM in our analysis and simulation in view of short-frame transmission, with their FFT sizes for both OFDM symbol generation and detection set as 2048 and 1024, respectively, including null subcarriers.

In the case of the 4-PAM, the first two bit position indices of $s$ for the type-1 (weaker odd-position) and the type-2 (weaker even-position) optimized block interleaver are specified by [2, 1] and [1, 2], respectively. Similarly, in the case of 16-PAM, we have [4, 1, 3, 2] and [1, 4, 2, 3] for the type-1 and the type-2, respectively.

A. BER Comparison of the Type-1 and the Type-2 Optimized Block Interleaver

When the convolutional code with $K_c = 3$ is used, there is only one error pattern $e_k(0) = [1, 1, 0, 1, 1, 1]$ at the minimum free distance $d_1 = 5$ as shown in Fig. 3, where the zero position is always matched with the odd position of the trellis segment. Therefore, the type-1 optimized block interleaver is expected to outperform the type-2 optimized block interleaver in this case. On the other hand, in the case of the convolutional code with $K_c = 7$, there are eleven error patterns at the minimum free distance $d_2 = 10$. By enumerating the number of zeros in each position of the trellis segments for all these error events (i.e., for $e_k(0), \cdots, e_{11}(0)$), it is found that zeros appear 61 times in the odd positions and 71 times in the even positions. Thus, the type-2 optimized block interleaver is expected to yield better performance than the type-1 optimized block interleaver for this case.

Figure 5 demonstrates the simulated BER performance of the type-1 and type-2 optimized block interleavers with the convolutional codes of $K_c = 3$ and 7 over an AWGN channel.

B. BER Comparison over an AWGN Channel

Figure 6 shows the comparison between the simulated BER performance and the approximate BER lower bound calculated by (39) over an AWGN channel. Since we employ the convolutional code with $K_c = 3$, the type-1 optimized block interleaver is employed as an optimized interleaver satisfying both $P_1$ and $P_2$. Among the three interleavers, the optimized block interleaver shows the best performance due to its smallest value of $P_{\text{min}}$. Simulated BER performance for BICM without interleaving is also shown for comparison.

Comparing the simulation results and the corresponding approximate lower bounds, in the low SNR, the gaps between them are large since only symbol errors to the nearest neighbor are taken into account in our analysis. On the other hand, in high SNR region, the approximate BER curves show good agreement with the simulation results. Therefore, we can conclude that the derived expression (39) can precisely estimate the performance of the BICM-OFDM system with a specific interleaver over an AWGN channel. Nevertheless, it is
Fig. 6. BER performance comparison of the simulation (sim) and the approximate lower bound (approx) for the convolutional code of $K_c = 3$ over an AWGN channel with the three interleavers; random, regular block, and type-1 optimized block interleavers. Simulated BER without interleaving is also shown as a reference.

Fig. 7. BER performance comparison of the simulation (sim) and approximate lower bound (approx) for the convolutional code of $K_c = 3$ over the frequency-selective Rayleigh fading channel with 15 independent equal-power taps with the three interleavers; random, regular block, and type-1 optimized block interleavers. Simulated BER without interleaving is also shown as a reference.

known that an insertion of any interleaver is suboptimal in the case of an AWGN channel since BICM without interleaving outperforms that with interleaving in asymptotically high SNR [20], thus suggesting that the proposed design guideline is still suboptimal for such a scenario.

C. BER Comparison over the Fading Channel

The BER performance comparison between the simulation and the corresponding theoretical analysis calculated by substituting (51) into (8) in the case of $K_c = 3$ over the frequency-selective Rayleigh fading channel with $L = 15$ equal-power taps is shown in Fig. 7. As a reference, the simulated BER in the case without bit interleaving is also shown. Similar to the case of an AWGN channel, the type-1 optimized block interleaver shows the best performance among the three interleavers. In particular, for the 16-PAM, the optimized block interleaver can obtain as much as 2.0 dB gain from the random interleaver. Thus, it can be concluded that the proposed design guideline is valid also for frequency-selective fading channels.

It is worth noting that contrary to the case of an AWGN channel, the BER performance without any bit interleaving is significantly inferior to those with interleavers since little diversity gain can be achieved when no bit interleaving is performed over frequency-selective fading channels. It can be thus concluded that the proposed design guideline based on the regular block interleaver is effective especially when the channel becomes dispersive.

The approximate lower bounds show good agreement with the simulated performance for the regular and type-1 optimized block interleavers. However, for the random interleaver, a noticeable gap is observed between the theoretical bounds and the simulated results. This is because the full diversity is assumed in our theoretical analysis even for the case of the random interleaver, whereas in practice, a permutation which cannot satisfy this condition may occur due to its random nature. In other words, the theoretical performance of the random interleaver presented here is the ideal case where the full diversity order is always guaranteed. The same argument applies to the superiority of the regular block interleaver over the random interleaver in the simulated BER performance for 4-PAM, even though the regular block interleaver has larger $P_{\text{min}}$ value than the random interleaver. Moreover, from the results of the type-1 optimized block interleaver, it can be concluded that under the block interleaving framework which effectively achieves the frequency diversity effect, the optimization through permutation of the regular block interleavers based on the error event vectors is a simple yet promising approach for frequency-selective fading channels.

VI. CONCLUSION

In this paper, we have derived the approximate BER lower bound expression for the convolutionally coded BICM-OFDM system with specific interleavers. Numerical results have shown that the approximation becomes tight when the SNR is high, and our analysis is valid for evaluating the performance with different interleaver structures. We have presented the design guideline for an optimal interleaver considering the distribution of the minimum Euclidean distance, and its effectiveness has been evaluated through the numerical results. We have also introduced the optimized block interleaver that satisfies the proposed design guideline while achieving full diversity effect and demonstrated its superiority over the conventional interleavers such as random and regular block interleavers.

REFERENCES


