# PEAK-POWER REDUCTION SCHEMES IN OFDM SYSTEMS: A REVIEW

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*Abstract* — **This paper reviews some of the peak-power reduction schemes that have been proposed for OFDM systems to date, and discusses their pros and cons.**

## I. Introduction

Because of a number of advantages over frequency-selective fading channels, Orthogonal Frequency-Division Multiplexing (OFDM) schemes [1] have proven to be a promising technique for broadcasting applications [2], and it is now being considered as a candidate for wireless LAN [3,4].

However, since OFDM signal consists of a sum of multiple tones, the waveform is inherently non-constant. This characteristic seriously limits their applications. If the signal with a large amplitude is input to high power amplifier (HPA) with nonlinear transformation, severe distortion occurs, resulting in both out-of-band radiation and in-band distortion. In-band distortion can be somewhat compensated by strong forward error control (FEC) techniques. Out-of-band radiation, however, causes serious adjacent channel interference (ACI), therefore typically restricted by regulations. If out-of-band radiation does not cause any problem, one may not need to care much about nonlinear distortion, and besides FEC, some compensation scheme at receiver, such as in [5], may be one possible solution to mitigate the degradation due to the in-band distortion. However, in the applications such as wireless LAN, out-of-band radiation *does* cause serious problems [4]. In order to reduce the out-of-band radiation as well as the in-band distortion, possible solutions may be:

- use of linear amplifier, or operate nonlinear amplifier with large back-off
- to reduce average power so as to reduce peak envelope power
- to reduce the peak to average power ratio (PAPR) of the signals

Linear amplifiers with high power efficiency around high radio frequency (several GHz) may call for state-of-the-art techniques. The operation of HPA with large back-off and the reduction of average power result in a severe power penalty. Hence, the most elegant way of out-of-band reduction may be the application of PAPR reduction techniques.

To date, various types of PAPR reduction techniques have been proposed. This paper reviews some of them, and discusses their pros and cons.

## II. Behavior of Peak-to-Average Power Ratio of OFDM Signals

The baseband expression of an OFDM signal with $N$ subcarriers (over one symbol, $T_s$) at the transmitter is

$$s(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{T_s} t} \tag{1}$$

where $X_k$ is a complex-valued data symbol of the $k$th subcarrier. Throughout the paper, we do not consider the effect of guard interval $T_g$ on the peak-power property.

Peak-to-average power ratio (PAPR) of the signal $s(t)$ is generally defined as [1]

$$\lambda = \mathrm{PAPR}[s(t)] = \frac{\max_{0\le t<T_s} |s(t)|^2}{\mathcal{E}[|s(t)|^2]} = \max_{0\le t<T_s} \frac{|s(t)|^2}{N} \tag{2}$$

where $\mathcal{E}[.]$ denotes expectation operation. It is easy to see that, for the worst case, theoretical maximum PAPR is N. Such probability is, however, practically zero for even relatively small $N$ (say, around 25)[25]. Therefore, we consider the statistical characteristic of PAPR. Assuming the amplitude of OFDM is subject to Rayleigh distribution, which becomes quite accurate for large $N$, the cumulative distribution function (cdf) of the PAPR $\lambda$ can be written as [10,14–16]

$$F(\lambda) = (1 - e^{-\lambda})^N, \tag{3}$$

and therefore probability density function (pdf) is

$$f(\lambda) = Ne^{-\lambda}(1 - e^{-\lambda})^{N-1}. \tag{4}$$

After some mathematical manipulation, one may obtain the mean value of $\lambda$

$$\mathcal{E}[\lambda] = \gamma + \psi(N + 1) \approx \gamma + \ln N \tag{5}$$

[1] Without any loss of generality, the (average) amplitude of each carrier is normalized to unity.

where $\gamma = 0.5772\ldots$ is the Euler constant and $\psi(x)$ is the digamma function. Variance of $\lambda$ can be also obtained as

$$\mathrm{VAR}[\lambda] = \frac{\pi^2}{6} - \psi'(N+1) \approx \frac{\pi^2}{6} \quad \text{(for large } N\text{)} \tag{6}$$

Even though the theoretical maximum PAPR increases linearly with $N$, the mean value increases only logarithmically. Hence, if we consider $N = 128$, the most likely (i.e., mean) PAPR value is only 7.4 dB (with variance 2.1 dB), which is far smaller than the theoretical maximum (21.1 dB). However, it is clear that PAPR of around 9 dB should be reckoned with for a practical PAPR estimation. The linear amplification of the signal with PAPR of 9 dB may be too large to be handled by HPA with reasonable power efficiency, and the peak-power reduction schemes may be necessary for many applications.

In what follows, we review some of these techniques recently proposed. Note that we exclusively consider the in-band and out-of-band distortions that arise within one OFDM symbol due to the nonlinearity of the channel. The out-of-band noises caused by an abrupt change of the symbols may be reduced simply by the use of appropriate windows [1].

## III. Peak-Power Reduction by Signal Distortion

First, we review PAPR reduction schemes with signal distortion. The primary aim of the signal distortion technique is to deliberately distort the transmission signals so as to suppress the out-of-band radiation[2]. These schemes can be realized at the cost of bit-error performance degradation, and do not require any additional redundancy.

### A. Signal clipping

The most straightforward solution for the peak-power reduction is deliberate clipping [7]. However, clipping itself is a nonlinear process, and it causes both in-band distortion and out-of-band radiation. Therefore, filtering is necessary to reduce the out-of-band radiation, as proposed in [8]. However, as mentioned in [8], filtering the clipped signals leads to peak-power re-growth, even though, as a whole, the resultant PAPR is likely to be smaller than the unclipped signals. The process itself is relatively simple, and applicable to any OFDM systems.

[2] In [6], the technique is proposed which uses two constant-envelope OFDM signals that compose one OFDM symbol, by adding some nonlinear functions to the original OFDM signals. This technique reduces in-band distortion, rather than out-of-band radiation, at the expense of the reduction of the transmission rate by half.

### B. Gaussian weighting

Pauli and Kuchenbecker [9] proposed more elaborate and effective approach by using the gaussian weighting function around the peak points. The peak-power-reduced transmission signal is obtained by

$$|s_{clip}(t)| = |b(t)| \cdot |s(t)| \tag{7}$$

where $b(t)$ is given by [9]

$$b(t) = 1 - \sum_{\substack{t_n \\ |s(t_n)|>A_{max}}} \left(1 - \frac{A_{max}}{|s(t_n)|}\right) g(t - t_n), \tag{8}$$

$t_n$ is the $n$-th position of the peak power exceeding some threshold $A_{max}$, and $g(t)$ is a gaussian function given as $g(t) = e^{-\alpha t^2}$, $\alpha$ denoting a coefficient which controls the out-band and in-band noises. This process is described in Fig.1, which shows that by weighting around peak points with a gaussian function, clipping becomes smooth, leading to the considerable reduction of out-of-band radiation. Moreover, by enlarging the width of the weighting function, i.e., by decreasing the coefficient $\alpha$, we may obtain smaller spectral spreading, at the cost of increased in-band distortion. Therefore, selection of the width of weighting function gives a trade-off between in-band and out-of-band noises. This scheme has been recently examined by several researchers [10,11]. In [10], this scheme was referred to as "peak windowing."

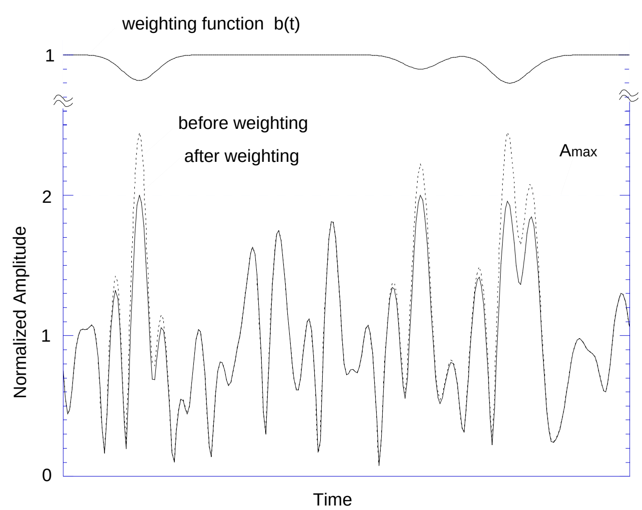


Fig. 1: Peak power reduction by weighting function ($N = 128$, $A_{max} = 2.0$)

## IV. Peak-Power Reduction by Redundant Signal Sets

In what follows, we describe the schemes that incorporate some redundancy in order to adaptively reduce the peak power of OFDM signals. Unlike the techniques described above, these schemes do not cause in-band distortion, at the cost of bandwidth efficiency.

### *A. Adaptive Selection of the Signals*

The simple, straightforward approach may be adaptive selection of several OFDM signals that represent the same information data, known as "phasor transformation" [12], "selective scrambling" [13], or "selected mapping" [14].

Let $\mathbf{X}$ denote the transmission data symbol vector, and $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{P-1}$ denote the transformed version of $\mathbf{X}$, all of which carry the same information as $\mathbf{X}$. Typically $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{P-1}$ are obtained by simply adding some phase rotations to $\mathbf{X}$. This system is depicted in Fig.2, where $f_p$ denotes the $p$-th transformation of the original signal. Then, the transmitter detects the peak power of each signals and transmit the signal with a minimum peak power. Note that the cdf of the selected signal can be written as

$$F_P(\lambda) = 1 - (1 - F(\lambda))^P, \qquad (9)$$

where $F(\lambda)$ is given in Eq.(3).

The redundancy required depends on the size of $P$, but in general relatively small. Computational complexity also depends on $P$, where the required devices are $P$ parallel FFTs and a peak detector. Note that the erroneous side-information leads to burst-error, so it must be protected with high priority.

In [15], the application of this scheme to OFDM-CDMA is reported.

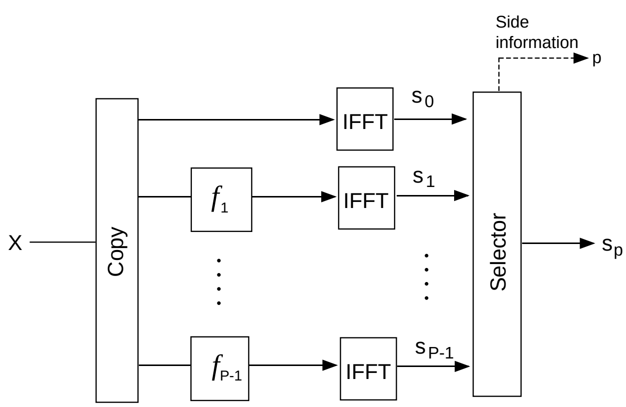


Fig. 2: An example of selection-based peak-power reduction schemes

### *B. Optimization of the Signals*

Another approach is the application of optimization processes, such as [16,17]. In these techniques, the transmission data vector $\mathbf{X}$ is first partitioned into several (say, $L$) blocks, $\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{L-1}$. Then, each block is subject to the transformation $f_l$ (see Fig.3). Optimization is performed over the set $\boldsymbol{f} = \{f_0, f_1, \ldots, f_{L-1}\}$ (in either time or frequency domain) such that the resultant signal $s(t)$ yields minimum PAPR, i.e., to choose $\boldsymbol{f}$ which minimizes the cost function

$$\mathrm{PAPR}\left[\sum_{i=0}^{L-1} f_i(\mathbf{X}_i)\right].$$

If the proper optimization is performed, these techniques offer much more PAPR reduction capabilities than the simple selective methods. Moreover, if differential detection over frequency domain is applied, the side information may not be required, with negligible redundancy. However, the price to be paid is the increased complexity – in many cases, optimal search of the transformation sets becomes challenging, forced to resort to sub-optimal approach, such as iterative algorithms, which may cause some transmission delay.

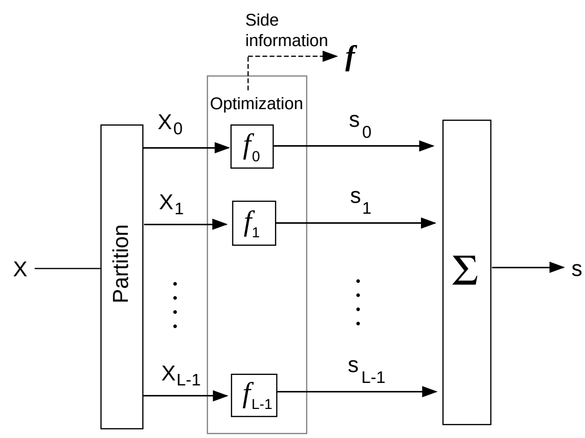


Fig. 3: An example of optimization-based peak-power reduction schemes

In [18], another variation of the optimization-based approach is proposed, which uses some additional (redundant) subcarriers to reduce the overall PAPR of the transmitted signals. Let $\mathbf{X}$ and $\mathbf{C}$ denote the transmission data vector and the additional peak suppression vector, respectively, of length $N$ (in frequency domain). Note that $\mathbf{C}$ consists of the redundant subcarriers, hence $\mathbf{X} \cdot \mathbf{C} = 0$, which means that $\mathbf{C}$ can be completely eliminated at the receiver without any help of side-information. The vector $\mathbf{C}$ must be chosen so as to minimize

$$\mathrm{PAPR}\,[\ \mathrm{IFFT}\ [\mathbf{X} + \mathbf{C}]].$$

In [18], the above function is further simplified such that the minimization can be carried out based on a Linear Program. Note that compared to the transformation methods, this scheme would require additional energy and bandwidth.

*C. Coding Techniques*

Basically, the optimization schemes require searching for optimum or near-optimum transformation or vectors, which leads to a large computational overhead at the transmitter. The other well-known are coding approach [19]. The basic idea of coding scheme is just to transmit the only sequences that yield very low PAPR. Consequently, the out-of-band and in-band distortion can be completely avoided by the use of HPA with moderate back-off. Some of these schemes require the brute-force code search, which is rendered to enormous computational effort [19,20].

On the other hand, some do not require any computational overhead. In particular, several researchers [23–25] independently propose coding schemes based on complementary sequences, which is capable of *both* peak-power reduction and forward error control. It is well known that the complementary sequences introduced by Golay [21] yield OFDM signal with PAPR always less than or equal to 3 dB [22]. Fortunately, systematic way of constructing Golay complementary sequences is known. Therefore, if one generates complementary sequences, then the peak-power reduction is fulfilled. Furthermore, the redundancy caused by the construction of complementary sequences can be efficiently used for error control.

In [25], for example, simple block coding approach is proposed under the following constraints; the modulation format is phase shift keying (PSK) and the number of carriers $N$ should be a power of 2.

Now, let us consider $M$-PSK OFDM. Complementary sequences $\mathbf{x} = \{x_0, x_1, \ldots, x_{N-1}\}$ (which is in integer modulo expression of $\mathbf{X}$) can be obtained by the following equation [25]

$$\mathbf{x} = \mathbf{u}\mathbf{G}_N + \mathbf{b}_N \pmod{M} \qquad (10)$$

where $N = 2^{K-1}$, $\mathbf{G}_N$ is a $K \times N$ matrix and $\mathbf{b}_N$ is a constant sequence of length $N$, both having $M$-ary integer elements. $\mathbf{u}$ corresponds to the $M$-ary information sequence of length $K$. $\mathbf{G}_N$ and $\mathbf{b}_N$ can be obtained by the simple recursive procedure [25].

The cost of the coding scheme is the resultant low coding rate; if PAPR is restricted to 3 dB, the coding rate is $R = (1 + \log_2 N)/N$, which turns out to be prohibitively low for large $N$. However, what should be noted is that we are not wasting the redundancy, and actually we are able to use it for an error control. The complexity of the transmitter is negligible, since PAPR reduction process is done by simple block encoding, i.e., Eq.(10). On the other hand, complexity of the receiver depends on the decoding algorithm. Fortunately, we can use some of the computationally efficient sub-optimal soft-decision decoding schemes [26], which have been developed basically for the conventional linear block codes, and brute-force search of code words can be avoided.

The union bound of the bit-error probability, $P_b$, can be expressed, after subtracting $\mathbf{b}_N$ from $\mathbf{x}_N$, as

$$P_b \leq \sum_{\text{all } \mathbf{x}} \frac{w_x}{N} Q\left(\sqrt{(log_2 M) R \frac{2E_b}{N_0} d_x}\right) \qquad (11)$$

where $w_x$ is a Hamming distance of the code word $\mathbf{x}$, $d_x$ is a projection of an Euclidean distance of $\mathbf{x}$ to one dimension, which is given by

$$d_x = \sum_{n=0}^{N-1} \sin^2\left(\frac{x_n}{M}\pi\right).$$

This bound is described in Fig.4, for $(N, K) = (8, 4)$ and $(16, 5)$ block codes with $M = 4$ and 8.

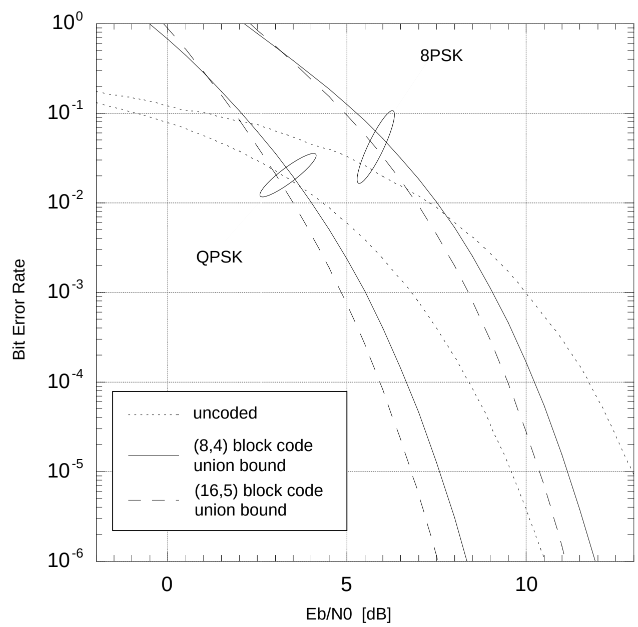


Fig. 4: Union bound of the block codes over AWGN channel

From the figure, despite the low coding rate, the coding gain may not be so significant, since the coding itself is not optimal, in an information theoretic sense. However, if the channel becomes frequency-selective, then this scheme offers considerable coding gain. To see this, some simulation result over typical indoor office environment is shown in Fig.5, where differential detection is applied so that no channel estimation is required. The parameters used in the simulation is summarized in Tab.1. Since the maximum delay spread well exceeds the guard interval, the error-floor due to intersymbol interference (ISI) is observed. However, with coding scheme, significant coding gain is achieved, in addition to the reduction

Tab. 1: System parameters for computer simulation with (16,5) block code ($N = 16$, PAPR=3 dB)

| | |
|---|---|
| Modulation | 4DPSK/8DPSK |
| OFDM symbol period $T_s$ | 1100 nsec |
| Guard interval $T_g$ | 300 nsec |
| Transmission data rate | 9.1 / 13.6 Mbps |
| Bandwidth efficiency | 0.43 / 0.64 |
| Maximum (rms) delay spread | 700 nsec (100 nsec) |

of the error-floor, due to the frequency diversity effect.

In [24], Davis and Jedwab pointed out the connection between Reed-Muller (RM) codes and complementary sequences, and suggested the decoding method based on RM codes. The approach based on RM codes offers some flexibility in the constraints of permissible PAPR and coding rates.

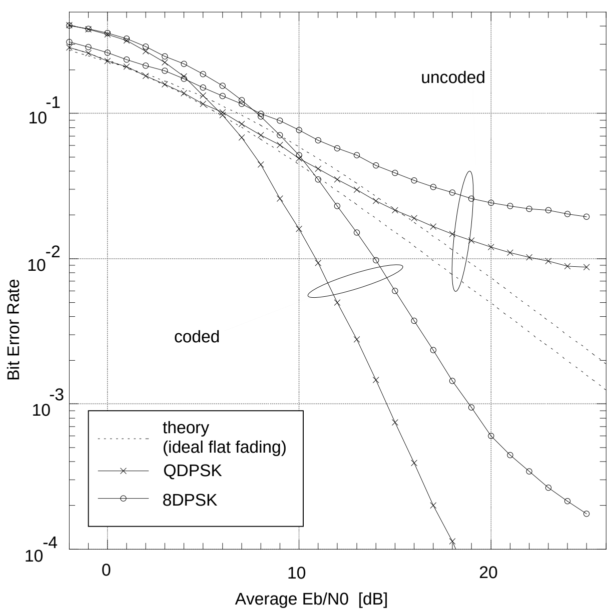


Fig. 5: Performance of the (16,5) block code over frequency-selective fading channel ($N = 16$, PAPR=3 dB)

## V. Discussion

As we have seen, if the number of subcarriers can be as small as 8 or 16, the coding approach may be attractive, since the PAPR reduction is guaranteed without any complicated computation, and the redundancy can be used to further improve the performance; after all, even though the required bandwidth expands, there would be no redundancy.

For OFDM systems with a relatively large number of subcarriers, it may be preferable to resort to other schemes. Peak-power reduction by signal distortion such as the use of weighting function may be a powerful solution for the out-of-radiation reduction, but at the cost of bit-error performance degradation. Even though the redundancy may not be required for that case, FEC must be necessary, which results in some redundancy any way.

On the other hand, peak-power reduction schemes by redundant signal sets require redundancy, even though it is relatively small compared to the transmission rate. However, system complexity may be of practical importance.

Furthermore, combination of these schemes may be also possible.

Finally, we show some results on the performance of the out-of-band-radiation reduction schemes over a nonlinear channel. As a memoryless nonlinear channel, we utilized the often-cited solid state power amplifier (SSPA) model of Rapp [27], of which the AM/AM conversion is given by

$$A_{out}(t) = \frac{A_{in}(t)}{\left(1 + \left(A_{in}(t)\right)^{2p}\right)^{1/2p}}, \qquad (12)$$

and AM/PM conversion is negligible. In the above equation, input amplitude $A_{in}(t)$ is normalized by the saturation amplitude, $A_{sat}$, and $p$ is the smoothing parameter. As a parameter of back-off, we use input back-off (IBO), which is defined as IBO $= A_{sat}^2/P_{in}$, where $P_{in}$ is the average power of the input signal.

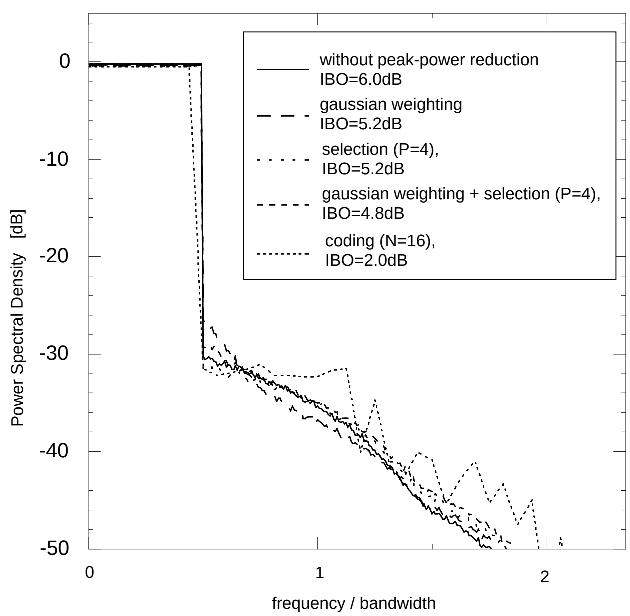


Fig. 6: Spectra of peak-reduced OFDM signals over SSPA channel ($p = 5$), $N = 128$ (except for coding $N = 16$ )

Following the analysis by van Nee and de Wild [10], we obtained Fig.6, which shows the required IBO in order to achieve power spectral density (PSD) with out-of-band radiation 30 dB below the main signal power. Note that in-band noise is not taken into consideration. With $N = 128$, the following four cases were examined; without any PAPR reduction, with gaussian weighting [9], with selection method [14] ($P = 4$), and with a combination of the both schemes. Obviously, the required back-off becomes smaller, as the complexity of the system increases. In Fig.6, PSD of the block coded signal with $N = 16$ is also shown for comparison. If coding scheme is used and in-band noise is tolerable, required IBO is only 2 dB. Moreover, since the maximum PAPR 3 dB is guaranteed, IBO of 3 dB may result in in-band and out-band noise free performance.

## VI. Conclusions

We have reviewed some of the PAPR reduction schemes recently proposed for OFDM signals, and discussed their applicability in terms of performance and complexity. Coding approach may be the most power-effective technique, but only available with a small number of subcarriers and low coding rate. Besides, a small-subcarrier system means that the overhead due to the guard interval becomes large, resulting in low bandwidth efficiency. In order to determine overall performance, further discussion may be required in terms of bandwidth efficiency and power efficiency.

The main focus in the paper was the out-of-band radiation problem, but the influence of the in-band noise must be also investigated.

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